# Lecture 10

1. Find local maximum/minimum for function of 2 variables

Let be a function with second-order derivatives exists. Then if we want to find its local maximum/minimum, we need to let , and such point satisfying is called a critical point.

However, we do not know whether a critical point is a max, min or neither. We need to look at second-order partial derivatives. Let Then

1. Justification:

For critical points, thus

(a) When the 2 terms in have the same sign, it is a local maximum or minimum. Thus if , it is minimum when , and maximum when .

(b) When the 2 terms in have the opposite sign, then it is a saddle.

(c) When , the , it depends on sign of .

* 1. .

(a) If need to look at third order terms

(b) If , it is a saddle

# Lecture 12

1. Gradient:
2. Tangent plane: on the tangent plane, Therefore, tangent plane is
3. is the direction in which the value of f increases fastest.

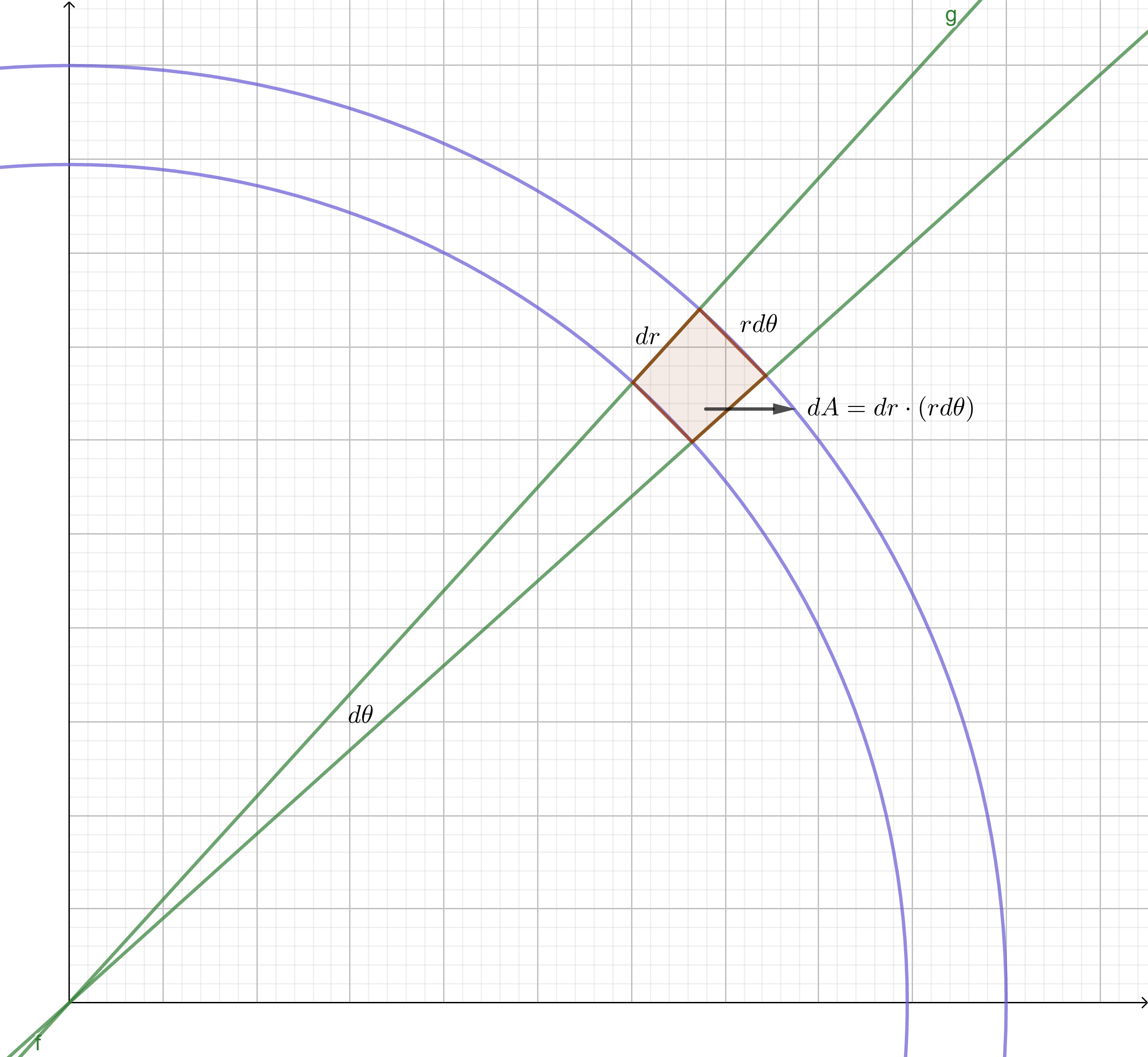
Assume is a unit vector, i.e., we can write the line in the direction of as

then the directional derivative is

When , i.e., and are towards the same direction, the directional derivative is maximized.

# Lecture 17 Double Integrals

1. Double integrals
2. Polar system:



# Lecture 18 Change of Variables in Double Integrals

1. Calculate the volume of the ellipse

Let then

1. If , then how do we compare and ?





Generally, if for some matrix , then

1. If

Then we have

Define Jacobian

We need the absolute value because area is non-negative.

1. Example: Evaluate , with change of variables

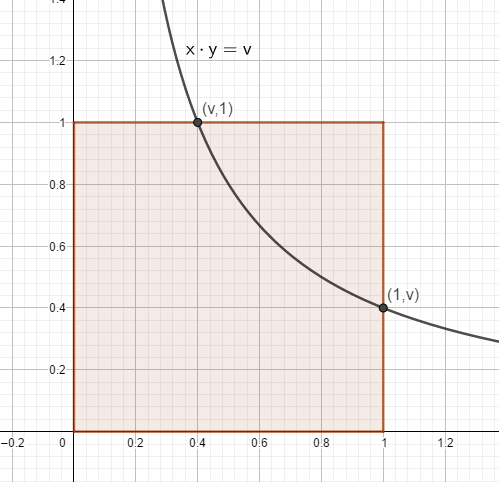
Method 1

Step 1: determine Jacobian

Step 2: determine the integrand

Step 3: determine the integral range

First determine the inner range . For the inner range, we keep as a constant. Then is on a hyperbola , and inside the square The range of is . Thus



The range of is the range of inside the unit square and should be .

Therefore

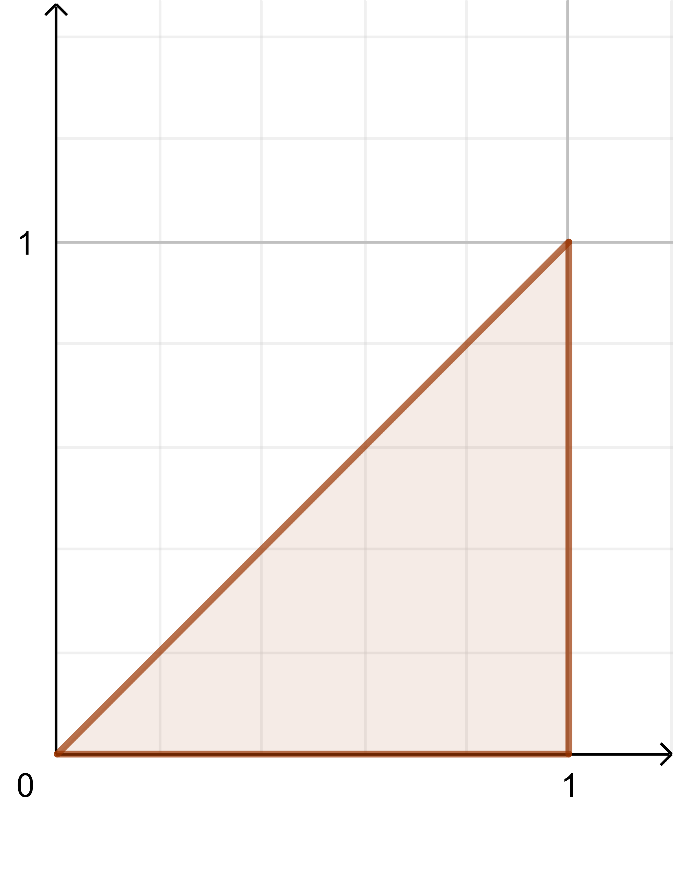
Method 2:

Step 1 and 2 are the same as method 1.

Step 3: The boundaries for are:

As we have:

Therefore, the integration range becomes:



# Lecture19 Line Integrals

1. Line integral: , where both and are vectors.

We cannot calculate something like , so we must find some relationship between and , e.g., express as , then

Also note that

1. Geometry way
2. Different notation of line integral

# Lecture 20-21 Conservative Field and Gradient Field

1. Fundamental theorem of calculus for line integrals:

Proof:

1. is called a conservative field, and it is path independent.
2. Three equivalent properties:
   1. is conservative
   2. is path independent
   3. is a gradient field
   4. is an exact differential
3. If is defined and differentiable everywhere, and , then is a gradient field.

# Lecture 22 Green’s Theorem

is a closed curve, enclosing a regioncounterclockwise, ands a vector field defined & differentiable in R, then

(Notice: R should be a simple connected region)

Proof: is a gradient field and .

Then we can assume N=0.

Cut into vertical simple pieces, and both sides of the equation equal of the sum of small pieces, so we only need to prove for small pieces.

The small pieces, with vertical sides, are easy to prove.

# Lecture 23 Flux

is a closed curve, enclosing a regioncounterclockwise, ands a vector field defined & differentiable in R, then the flux of is

where is the normal (perpendicular) unit vector along , specifically, we rotate the tangent direction 90 degrees clockwise to get , and

If , then

Comparison with work:

* Work in line integral along the curve, while flux is line integral perpendicular to the curve.

# Lecture 25-26

1. Cylindrical coordinates
2. Spherical coordinates